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NONLINEAR TIME-DOMAIN MODELS OF HUMAN CONTROLLERS

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INTRODUCTION

Analysis of the stability and performance of control systems in which a human controller is an active element has been hampered by the lack of an adequate mathematical model of the human control function. The recognized pioneer in the problem of determining models of human controllers was Tustin (ref. 1), who, in 1947, reported on compensatory tracking experiments and used the data from these experiments to formulate a model of a human controller. Many investigators since Tustin have analyzed data from similar experiments to formulate human control-response models. Because the human controller or pilot flying an airplane adapts or changes his technique as the dynamics of the plant or airplane changes, many experiments are necessary. In the compensatory tracking experiment (see fig. 1) the pilot is asked to minimize the error, e, displayed to him by an oscilloscope, television screen, or meter by manipulating a controller. The controller deflection, c, is sent to an analog computer which computes the response of the controlled element and adds to it the input disturbance function, i, forming an error which, in turn, is sent to the display. The signals are either processed during the experiment or recordings are made of the signals and later processed to obtain the model of the pilot (ref. 2). Similar experiments have been performed in flight in which the pilot maneuvers the airplane (refs. 3 and 4).

Data resulting from such experiments have been analyzed, and linear pilot models have been obtained (refs. 2 to 4) for a limited set of controlled-element dynamics. The methods used to construct the pilot models have been almost exclusively in the frequency domain (ref. 2). Recently, the time-domain analysis (ref. 5) has been applied to the problem of modeling pilots. Almost all the analysis of human control response that has been performed, however, has been in terms of linear models because human control response is often almost linear and the linear analysis is simpler. Figure 2 shows a block diagram of such a model. The part of the human controller's response that deviates from a linear, constant-coefficient model is represented by a noise signal, r, added to the output of the linear model in the manner shown.

Most of the small amount of nonlinear analysis that has been performed has been ad hoc in the sense that specific nonlinearities have been assumed and their characteristics determined by manual adjustment (ref. 6) or by

least squares (ref. 7). The time-domain analysis of Balakrishnan (ref. 8) offers a means by which nonlinear systems can be analyzed and modeled without having to assume specific nonlinearities. The only assumptions necessary are those that pertain to a Volterra integral series expansion. This generality is particularly important when little is known about the system being modeled, as with human controllers.

The initial results of the nonlinear time-domain analysis were reported in reference 5. Since that time, additional analyses have been made. This paper presents some of the results of these subsequent analyses and discusses the method of selecting the maximum memory time and the order of the nonlinear model. In addition, some results of orthogonal expansion of the weighting functions for reasons of data compression and reduction computation are presented and discussed.

LINEAR TIME-DOMAIN METHOD

Let us now consider a linear analysis in the time domain in which the output of a linear pilot model is expressed in the form (see ref. 8)

$$c(t) = \int_0^{T_M} h_p(\tau)e(t - \tau)d\tau + r(t)$$

Because the time histories $c(t)$ and $e(t)$ must be sampled for analysis, it is more appropriate to write

$$c(n) = \sum_{m=1}^{M} h_p(m)e(n - m + 1) + r(n)$$

or in matrix form

$$\underline{c} = E\underline{h}_p + \underline{r}$$

where

$$E = \begin{bmatrix} e(M) & e(M-1) & \ldots & e(3) & e(2) & e(1) \\ \cdot & & & e(4) & e(3) & e(2) \\ \cdot & & & & e(4) & e(3) \\ \cdot & & & & & e(4) \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ e(N-1) & & & & & e(N-M) \\ e(N) & e(N-1) & \ldots & & & e(N-M+1) \end{bmatrix}$$

$$\underline{h}_p = \begin{pmatrix} h_p(1) \\ h_p(2) \\ \cdot \\ \cdot \\ h_p(M) \end{pmatrix}$$

$$\underline{c} = \begin{pmatrix} c(M) \\ c(M+1) \\ \cdot \\ \cdot \\ c(N) \end{pmatrix} \qquad \underline{r} = \begin{pmatrix} r(M) \\ r(M+1) \\ \cdot \\ \cdot \\ r(N) \end{pmatrix}$$

The sampled impulse response of the pilot model, $h_p(m)$, can be obtained by using the least-squares formulation

$$\hat{\underline{h}}_p = \left[E^T E\right]^{-1} E^T \underline{c}$$

Inherent in the time-domain representation of the pilot model is the assumption that the output at any one time is a function of only a finite time of the history of the error. This maximum memory is denoted by M in the expression of the pilot model output. Figure 3 shows a typical result of such an analysis. It can be seen that the model impulse response first peaks at about 0.3 second, then oscillates as it subsides to zero. The oscillation indicated in the pilot's impulse response function is typical and is thought to be due to the dynamics of the combination of the control stick and the pilot's arm.

The time-domain results can be transformed to the frequency domain for comparison with the frequency-domain results through the use of the Fourier transform so that

$$\hat{Y}_p(j\omega) = F\left[\hat{h}_p(\tau)\right]$$

Figure 4 shows such a comparison in terms of amplitude ratio and phase angle. Although there is fair agreement between the time-domain and frequency-domain results, considerably less variance is evident in the time-domain results, as is indicated by the smaller range of values determined from three independent sets of data. The variance in the frequency-domain results is particularly severe at the lower frequencies where the error, which is the input to the pilot, is kept very small. For frequency-domain analysis, an input disturbance must be used which is "large" compared with the remnant or noise of the pilot. If this is not true, the measured pilot model becomes the inverse of the transfer function of the plant or airplane. This does not apply

in the time-domain analysis, however, because only causal or realizable models result from the requirement that the model response follow the input. Wingrove and Edwards (ref. 9) of the Ames Research Center have in fact analyzed, using the time-domain method, flight data for which no disturbance input was present except that due to the pilot's remnant. If such a procedure proves to be generally applicable, special tracking experiments will not always be required and it will be possible to use data heretofore unanalyzed to determine pilot models by means of the time-domain method of analysis. Still another advantage of analysis in the time domain is the capability of constructing nonlinear pilot models.

NONLINEAR TIME-DOMAIN METHOD

Nonlinear behavior on the part of the pilot accounts for at least part of the remnant of a linear pilot model. It is, therefore, of interest to investigate nonlinear pilot models. The formulation of a nonlinear time-domain pilot model can be expressed by using a Volterra integral series:

$$c(t) = \int_0^{T_M} h_{p_1}(\tau)e(t-\tau)d\tau$$

(linear)

$$+\int_0^{T_M}\int_0^{T_M} h_{p_2}(\tau_1,\tau_2)e(t-\tau_1)e(t-\tau_2)d\tau_1 d\tau_2$$

(quadratic)

$$+\int_0^{T_M}\int_0^{T_M}\int_0^{T_M} h_{p_3}(\tau_1,\tau_2,\tau_3)e(t-\tau_1)e(t-\tau_2)e(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + r(t)$$

(cubic)

or in the discrete case

$$c(n) = \sum_{m=1}^{M} h_1(m)e(n-m+1)$$

(linear)

$$+\sum_{m_1=1}^{M}\sum_{m_2=1}^{M} h_2(m_1,m_2)e(n-m_1+1)e(n-m_2+1)$$

(quadratic)

$$+ \sum_{m_1=1}^{M} \sum_{m_2=1}^{M} \sum_{m_3=1}^{M} h_3(m_1, m_2, m_3) e(n - m_1 + 1) e(n - m_2 + 1) e(n - m_3 + 1)$$

(cubic)

$$+ \underbrace{\ldots\ldots\ldots\ldots\ldots}_{\text{(higher order)}} + r(n)$$

As for the linear example, these expressions can be easily put in terms of matrices.

If again

$$\underline{c} = \begin{pmatrix} c(M) \\ c(M+1) \\ \vdots \\ c(N) \end{pmatrix} \qquad \underline{r} = \begin{pmatrix} r(M) \\ r(M+1) \\ \vdots \\ r(N) \end{pmatrix}$$

but h is expanded to include the elements of the higher order weighting functions so that

$$h = \begin{pmatrix} h_1(1) \\ h_1(2) \\ \vdots \\ h_1(M) \\ h_2(1,1) \\ h_2(1,2) \\ \vdots \\ h_2(1,M_2) \\ h_2(2,2) \\ \vdots \\ h_2(M_2,M_2) \\ h_3(1,1,1) \\ h_3(1,1,2) \\ \vdots \\ h_3(1,1,M_3) \\ h_3(1,2,2) \\ \vdots \\ h_3(M_3,M_3,M_3) \end{pmatrix}$$

and if E is expanded in a similar way so that

$$E = \begin{bmatrix} e(M_1) \ldots e(1)e(M_2)^2 \ldots (e(M_2)e(1)) \ldots e(1)^2 e(M_3)^3 \ldots\ldots e(1)^3 \\ e(M_1+1) \ldots e(2) \quad \ldots \\ e(M_1+2) \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ \ldots \quad \ldots \\ e(N) \ldots e(N-M_1+1)e(N)^2 \ldots\ldots e(N-M_2+1)^2 \ldots e(N-M_3+1)^3 \end{bmatrix}$$

we can again write

$$\underline{c} = E\underline{\hat{h}} + \underline{r}$$

and

$$\underline{\hat{h}} = \left[E^T E\right]^{-1} E^T \underline{c}$$

It is difficult to present the results of such an analysis in a meaningful form, but it is instructive to look at an example step response. Figure 5 shows (1) the response of the linear portion of the model and (2) the total response of the nonlinear model. A step input was used with an amplitude equal to twice the root-mean-square of the error or input to the pilot. The responses have been normalized for comparison. As the amplitude of the input is reduced, the response will approach that shown for only the linear portion. The only significant difference is the greater overshoot for larger

inputs and a slight increase in gain (evidenced by the larger steady-state response) of the larger input.

SELECTION OF MEMORY TIME AND ORDER

One method of assessing the worth of a model is to consider the fit error or the mean square of the difference between the measured and the calculated response. In figure 6 the fit error, which has been normalized by dividing by the near square of the total response, is plotted against both the maximum memory time on the left and the order of the nonlinearity of the model on the right. It is apparent that the fit error continues to decrease as either the memory time or the order is increased. It is of course not surprising that the fit error is reduced, since both increased memory time and increased order result in more degrees of freedom for the model. If as many elements in the model are allowed as there are data points, the fit error would be zero.

Another consideration is the variance or lack of certainty with which the weighting function elements can be determined. In figure 7 a lower bound of the average variance of the weighting elements is plotted against the same quantities, the maximum memory time and the order of the model. The estimates of the variance are based on the Cramer-Rao inequality in the same manner as was done by Astrom in reference 10. The values have been arbitrarily normalized. It is evident that the uncertainty of the weighting function increases as the memory time or order increases. Consequently, the selection of memory time and order should consider both the fit error and the variance.

Since we will use a model to predict the pilot's response for an unknown input, let us next consider the fit error, not for the same data used to determine the model but for an independent or "new" set of data. Figure 8 shows that the fit error now increases with increased order. On the basis of this information, one would conclude that a linear model with a maximum memory exceeding 5 seconds should be used. This result should serve as a warning against believing that a reduction of the fit error over the same data to define the model is necessarily an improvement.

The large difference in the fit error that results from using the same and new data indicates that either more data or a longer run length is needed. An example of the effect of run length on the difference of the fit error is presented in figure 9. It can be shown that the expected value of the fit error for an infinite amount of data is approximately the average of the values for the same and new data. The run length of 1 minute that was used in figures 6, 7, and 8 is inadequate to accurately determine the expected value of the fit error. Consequently, the run length was increased to 4 minutes. Figure 10 shows the final fit-error results. There is no discernible decrease in fit error for values of memory time in excess of about 3.25 seconds for a linear model. The reduction in fit error that results from using a third-order nonlinear model as compared to a linear model is 4 percent out of 36 percent. It is not expected that this reduction would warrant the added complexity of a

nonlinear model in most applications. Although it cannot be said with certainty that appreciable further reductions in the fit error cannot be made by using even higher-order nonlinear models, the indication is that higher order models are not warranted. This would indicate the bulk of the remnant to be stochastic rather than deterministic, a result consistent with results reported in reference 5.

DATA COMPRESSION

One problem that faces the analyst in a comprehensive study is the difficulty of summarizing the results of perhaps a hundred cases each having as many as 55 weighting elements. One approach to the problem for linear models has been to use 10 terms of a LeGuerre polynomial expansion (refs. 11 and 12). This represents a reduction from about 20 quantities (gain and phase) needed for characterization in the frequency domain to 10 coefficients for the LeGuerre polynomial representation of the impulse response function. One wonders, however, if there are not better functions to be used, especially for nonlinear models. A method suggested by Dr. A. V. Balakrishnan, of the University of California at Los Angeles, and motivated by the spectral representation of the information matrix appears to answer this question. A large, representative collection of weighting functions is used to form a summation of outer products

$$S = \sum_{i=1}^{N} h_i h_i^T$$

The Eigen values and the corresponding Eigen functions of the matrix are then determined. The Eigen vectors which correspond to the principal Eigen values can then be used as functions for expanding the weighting functions. This method was applied to a collection of linear weighting functions with satisfactory results. Only 3 or 4 of the Eigen vectors were necessary to characterize 38 weighting functions, each with 13 elements. The principal Eigen vector is plotted in figure 11 and appears to be a typical impulse response, as would be expected. Only 3 or 4 values are now needed to characterize the model which in the frequency domain required 20. The potential savings in data and in reduced computation for nonlinear models is even greater since the modeling problem reduces to determining a few to several coefficients as opposed to determining a much larger number of weighting elements.

CONCLUDING REMARKS

Analysis in the time domain is more advantageous than analysis in the frequency domain because (1) fewer values are needed for a characterization (this is especially true if the proposed method expansion for the weighting function is used) and (2) greater accuracy is achieved, especially when the input disturbance is not large compared with the remnant.

The fit error over the data used to determine the model decreases with increased memory time and order of nonlinearity. This result can be misleading, however, as the average variance of the weighting terms increases. The fit error over new or independent data should be used to assess which memory time and what order of nonlinearity should be used.

Eigen vectors that correspond to the principal Eigen values of a summation matrix of the outer products of a large, representative sample of weighting functions have proved to provide an efficient orthogonal expansion. The determination of the coefficients of these functions, as opposed to that of a much larger number of weighting-function elements, promises to offer not only a means of summarizing large sets of results but also of reducing the computation necessary.

Although the results from applying nonlinear time-domain analysis to the problem of modeling the human controller have been useful, any advantage of a nonlinear model over a linear model or the human controller performing a compensatory tracking task appears to be small. This result would not have been known, however, if nonlinear models of the human controller had not been made.

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SYMBOLS

c	pilot output (control deflection), inches
E	error matrix
e	error, radians
$F[\ ]$	Fourier transform
h	time interval, seconds
h_i	sample of impulse response of pilot
h_p	impulse response of pilot, inches/radian or inches/degree
i	input (external disturbance function), radians
M	maximum value of m, $M = \frac{T_M}{\Delta\tau}$
m	index for the argument of h_p
N	maximum value of n
n	index for time
o	linear output of pilot model (control deflection), inches
r	remnant signal of pilot model (control deflection), inches
S	matrix
s	Laplace variable
T_M	maximum memory time of the pilot model, seconds
t	time, seconds
Y_c	transfer function of controlled element
$Y_c(j\omega)$	controlled-element transfer function, radians/inch
$Y_p(j\omega)$	pilot describing function, inches/radian
τ	argument of h_p, seconds
$\Delta\tau$	incremental value of τ, seconds
ω	frequency, radians/second

$\wedge$	estimate
$\lvert\ \rvert$	absolute value
$\angle$	phase angle

Matrix notation:

(x), $\underline{x}$	column matrix
$[X]$	rectangular or square matrix
X^T	transpose
X^{-1}	inverse

Numbers used as subscripts denote the pertinent term or terms of the Volterra integral series or summation.

A COMPENSATORY TRACKING TASK

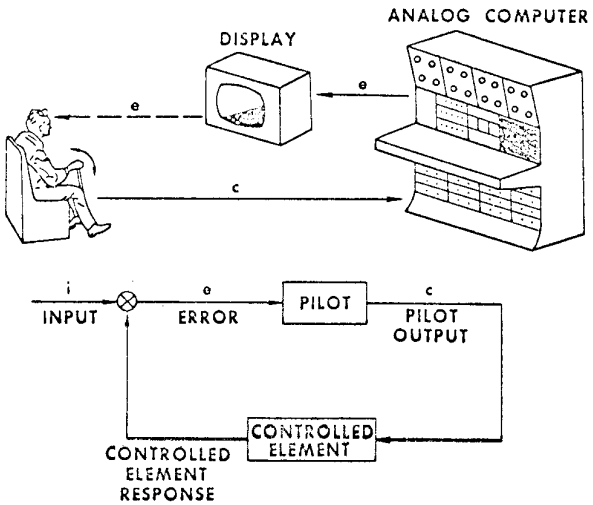

Figure 1

FREQUENCY DOMAIN MODEL OF A PILOT

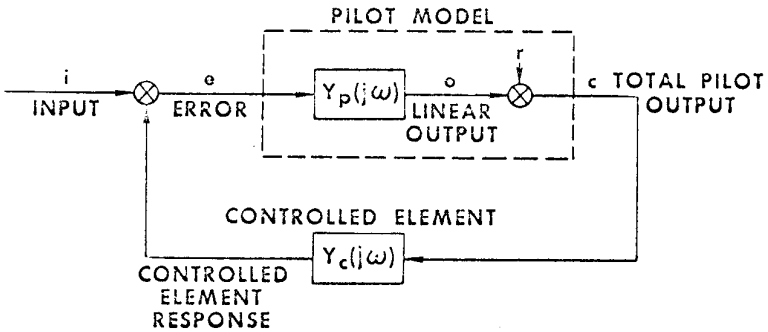

Figure 2

PILOT MODEL IMPULSE RESPONSE FUNCTION

$Y_c = 44/s$

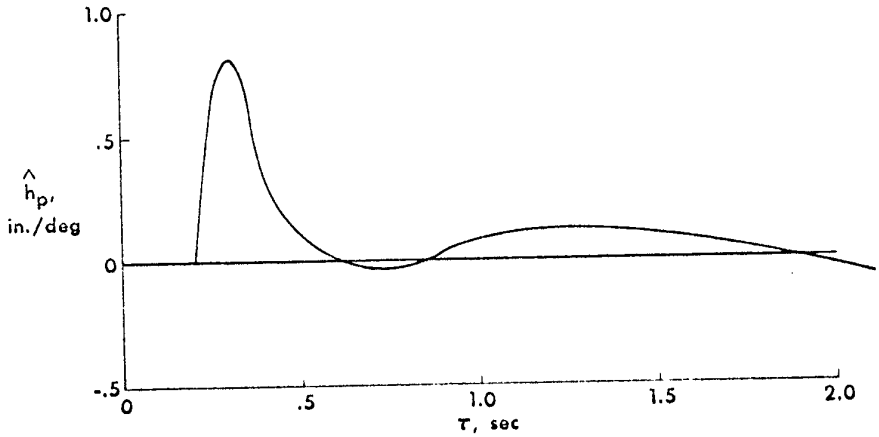

Figure 3

A COMPARISON OF TIME AND FREQUENCY DOMAIN RESULTS

$Y_c = 44/s$

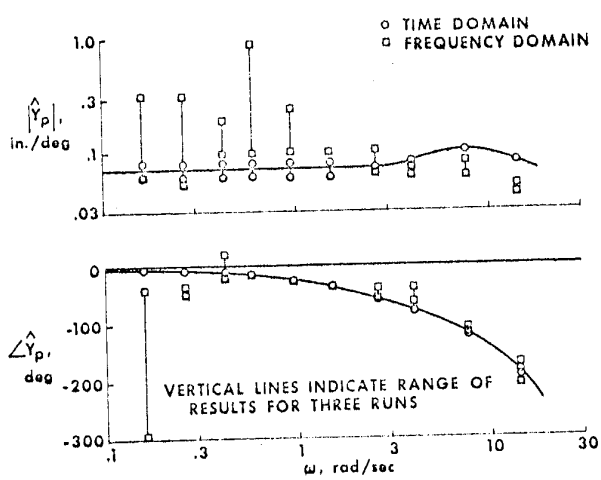

Figure 4

COMPARISON OF STEP RESPONSES FOR A LINEAR AND A NONLINEAR PILOT MODEL

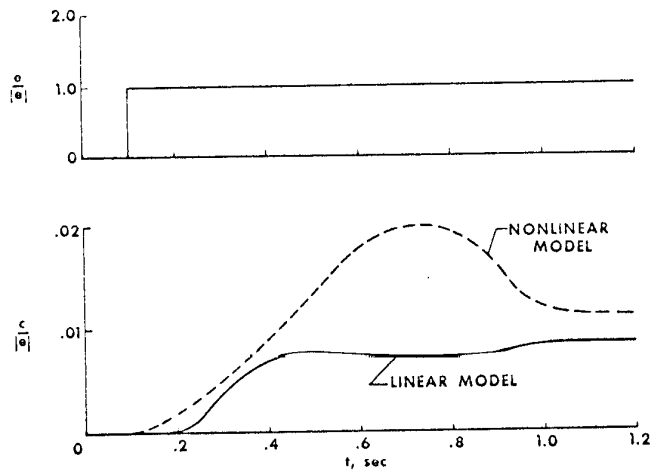

Figure 5

FIT ERROR AS A FUNCTION OF MAXIMUM MEMORY TIME AND ORDER OF MODEL

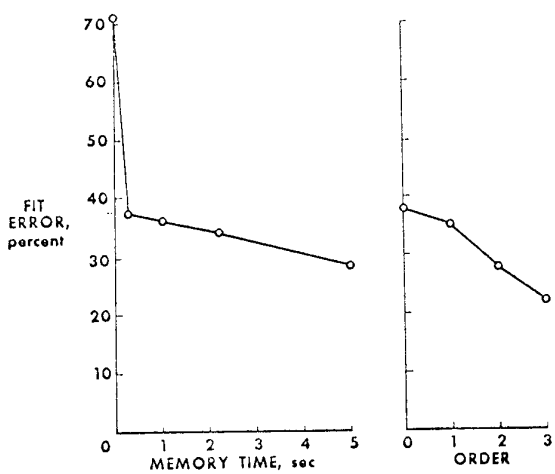

Figure 6

AVERAGE VARIANCE OF WEIGHTING TERMS AS A FUNCTION OF MEMORY TIME AND ORDER

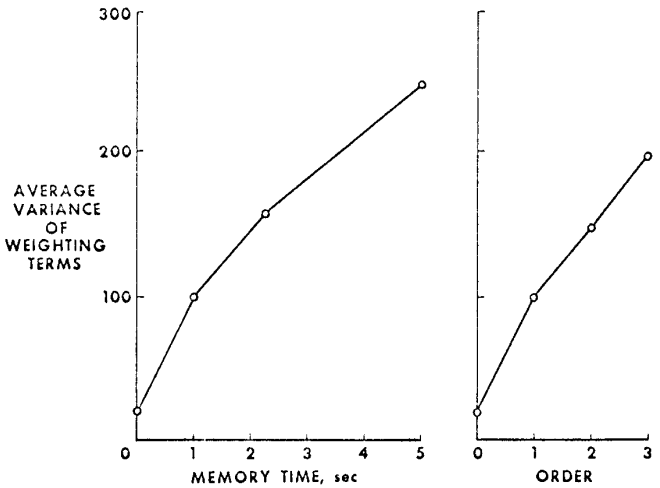

Figure 7

FIT ERROR USING SAME AND NEW DATA

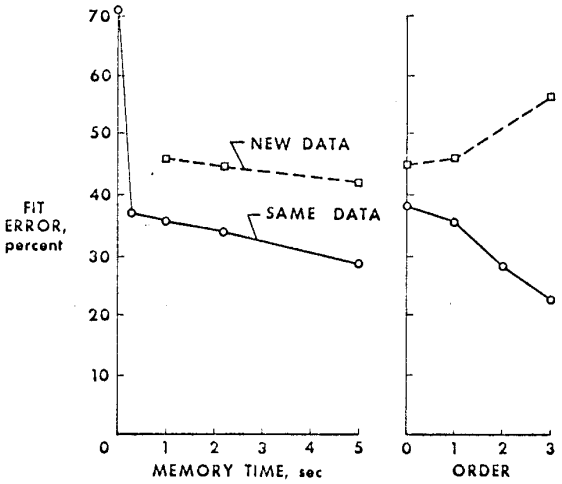

Figure 8

EFFECT OF RUN LENGTH ON FIT ERROR

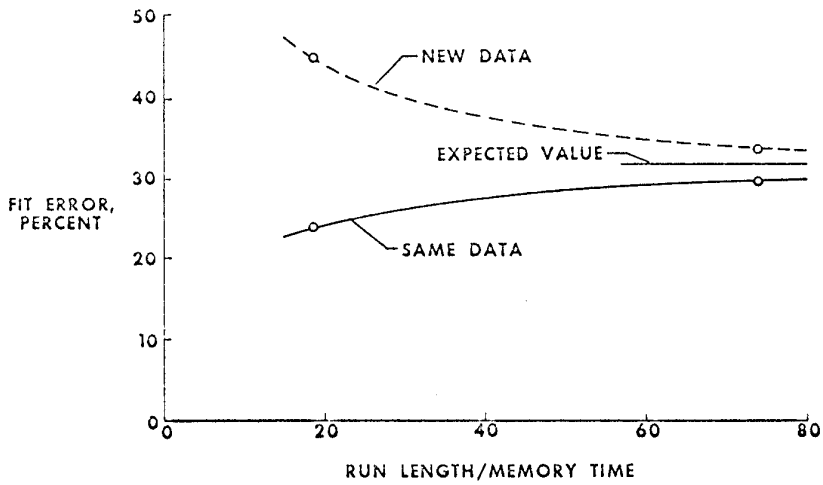

Figure 9

EXPECTED VALUE OF FIT ERROR

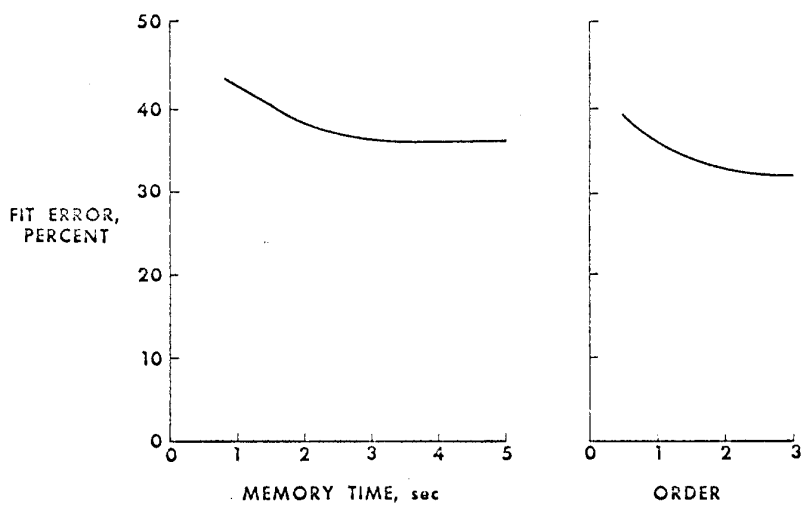

Figure 10

FIRST EIGEN FUNCTION

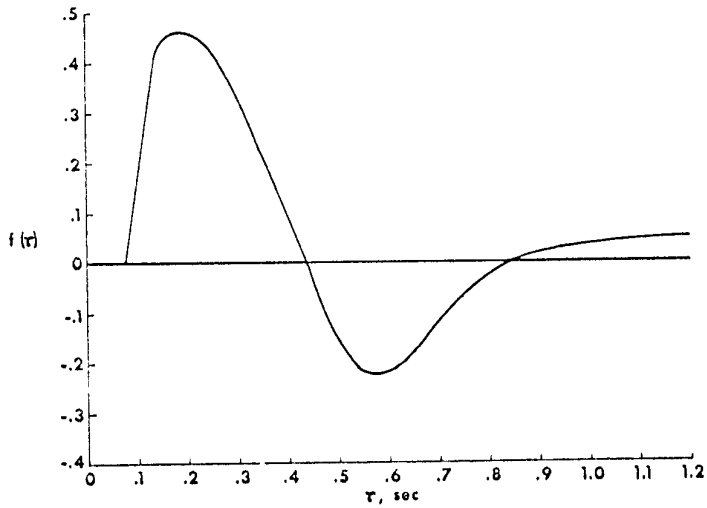

Figure 11