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A Presentation on Robust Flutter Margin Analysis and a Flutterometer

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October 1997

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ABSTRACT

This paper documents an invited presentation given to The Boeing Company, Seattle, Washington on September 9, 1997. The audience consisted of structural dynamic and flight test engineers from the Boeing Commercial Airplane Group who were interested in discussing research which may be applied to future flight flutter test programs. A method to compute robust flutter margins is described which is a significant departure from traditional methods. This method uses the structured singular value, μ, to compute a flutter margin which directly accounts for modeling errors such that a worst-case flutter margin is computed with respect to those errors. This method may be applied in several ways. A post-flight application uses data sets from multiple test points to compute worst-case flutter margins and a worst-case flight envelope. An on-line implementation computes flutter margins at each test point to track the flutter margins during a flight test. This on-line implementation is the basis for a flutterometer flight test tool that displays the distance to flutter at a given test point. Such a tool was not previously possible using traditional flutter flight test analysis methods. The F/A-18 System Research Aircraft was used to demonstrate these applications using flight data recorded from test points throughout the flight envelope.

NOMENCLATURE

C	damping matrix
K	stiffness matrix
M	mass matrix
P	plant transfer function
$\mathcal{P}$	set of plant transfer functions
$\bar{q}$	dynamic pressure
Q	unsteady aerodynamic force matrix
s	Laplace variable
β	pole in Padé approximation to a lag
$\delta_{\bar{q}}$	perturbation to dynamic pressure
Δ	uncertainty operators
$\mathbf{\Delta}$	set of uncertainty operators
Δ_β	uncertainty in β
η	state vector
μ	structured singular value
$\mathcal{H}_\infty$	set of stable, linear time-invariant transfer functions
$\mathcal{L}_2$	set of finite square-integrable measurements

μ Method : Research Team at Dryden

- **Marty Brenner**
  - Initiated research program
  - Main partner on applying robustness to aeroelasticity
- **Len Voelker**
  - Formulated initial concept for flutterometer
  - Computed *p-k* flutter analysis to compare with μ method
  - No living human has more practical knowledge of flutter
- **Larry Freudinger**
  - Developed on-line implementation concepts
- **Dave Voracek**
  - Chief engineer on F/A-18 SRA who generated and analyzed data
- **Roger Truax**
  - Generated finite element model of F/A-18 SRA
- **Mike Kehoe**
  - Supervised team efforts and considered practicality issues

The structural dynamics group at NASA Dryden Flight Research Center has been actively involved in this research project for some years. The diversity of the team enables significant research issues to be addressed by engineers with specialization in that area.

μ Method : Publications

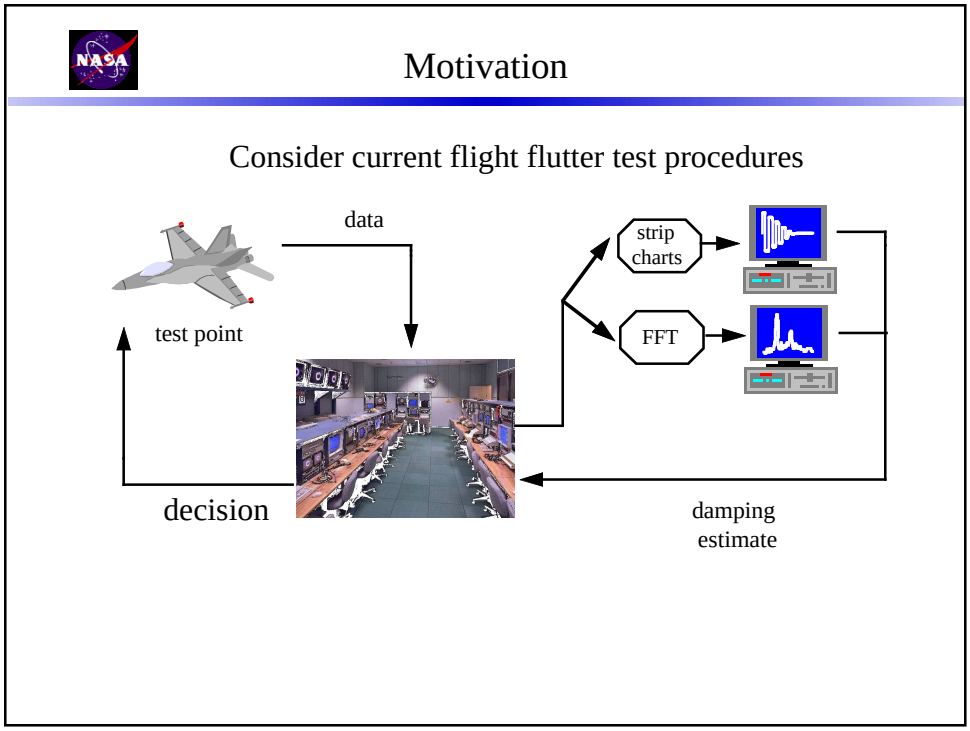
- R. Lind and M. Brenner, *Robust Flutter Margin Analysis that Incorporates Flight Data*, NASA Technical Paper, in preparation.
- R. Lind and M. Brenner, "Robust Flutter Margins of an F/A-18 Aircraft from Aeroelastic Flight Data", *AIAA Journal of Guidance, Control and Dynamics*, Vol. 20, No. 3, May-June 1997, pp. 597-604.
- R. Lind and M. Brenner, "Utilizing Flight Data to Update Aeroelastic Stability Estimates", *1997 AIAA Atmospheric Flight Mechanics Conference*, New Orleans LA, AIAA-97-3714, August 1997.
- R. Lind and M. Brenner, "A Worst-Case Approach for On-Line Flutter Prediction" *International Forum on Aeroelasticity and Structural Dynamics*, Rome Italy, June 1997, Vol. 2, pp. 79-86.
- R. Lind, M. Brenner and L. Freudinger, "Improved Flight Test Procedures for Flutter Clearance" *International Forum on Aeroelasticity and Structural Dynamics*, Rome Italy June 1997, Vol. 3, pp. 291-298.
- R. Lind and M. Brenner, "Worst-Case Flutter Margins from F/A-18 Aircraft Aeroelastic Data" *1997 AIAA Structures, Structural Dynamics and Materials Conference*, Orlando FL, AIAA-97-1266, pp. 738-748.

There are several refereed publications which discuss aspects of the method for robust flutter margin analysis. The NASA Technical Paper, which is expected to appear in early 1998, is the most detailed and complete reference document.

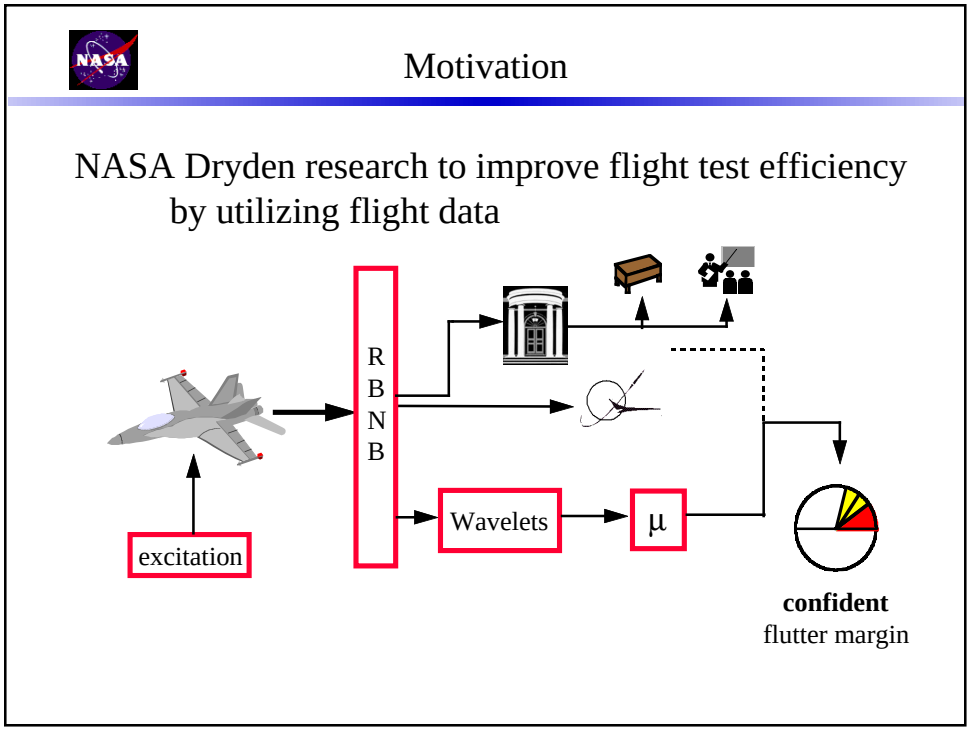
Outline

- Motivation
- μ Method for Robust Flutter Margin Analysis
  - Concept of robustness and uncertainty for flutter margins
  - Incorporating flight data
- Worst-Case Flutter Margins : Post-Flight Analysis
  - Implementation and approaches to utilize flight data
  - Application : F/A-18 SRA
- Flutterometer : On-Line Analysis
  - Implementation and approaches to utilize flight data
  - Application : F/A-18 SRA
- Conclusions
  - Research extensions
  - Open discussions

This presentation has essentially three large sections. Some initial comments are given to briefly discuss flutter analysis issues and provide the motivation for this research. The main section introduces the method for flutter analysis, called the μ method, which produces robust flutter margins. This portion of the presentation details the issues of robustness and demonstrates their applicability to flutter analysis. The other large sections describe implementations of the μ method and demonstrate these implementations on the F/A-18 Systems Research Aircraft. The first implementation is a post-flight analysis which computes worst-case flutter margins to define a flight envelope. The second implementation is the on-line tracking of robust flutter margins via the flutterometer concept.



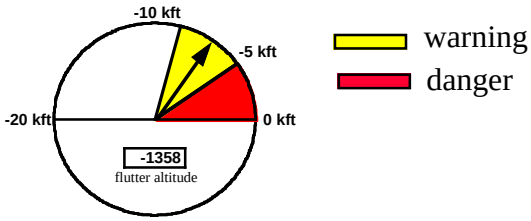
The procedures for flight flutter testing to clear a flight envelope are fairly standard throughout NASA and the flight test community, although implementation details vary widely between organizations. Response data is recorded from the aircraft at a stabilized test point and telemetered to the control room. The data is analyzed using strip chart monitoring and several computational algorithms in both the time and frequency domains. Estimates of the modal damping ratios and trends for those ratios as the envelope is expanded are used to determine the next suitable test point which does not incur an excessive level of risk.



NASA Dryden Flight Research Center is researching methods to improve flight test efficiency. Increasing efficiency implies reducing time and cost along with maintaining a high level of safety for the aircraft and crew. The entire scope of flight flutter testing is being investigated. The first step in the process investigates better signals and mechanisms to excite the aircraft and improve the dynamics observed with response data. An RBNB concept is being implemented to distribute this data to a variety of analysis facilities. Improved signal processing algorithms such as wavelet filtering are being utilized to accurately process transient response data. The last step in the process is to analyze this data to produce a confident flutter margin rather than the damping estimate which is currently computed. This last step is the focus of this presentation and is addressed by a parameter called μ.

Motivation

- Introduce concept of a **flutterometer**
  - flight test tool
  - indicates distance to flutter



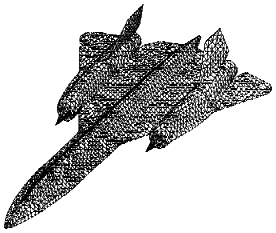
- Information about flight condition at flutter
  - altitude
  - dynamic pressure
  - airspeed

These research areas are all steps towards the concept of a flutterometer which was envisioned in the 1980's at NASA Dryden Flight Research Center. This flutterometer is a flight test tool that indicates some measure of distance to a flutter condition. The center box in the dial present the altitude at which flutter occurs. The values along the dial present the difference between that altitude and the current altitude at which the aircraft is flying.

The type of measure, such as altitude or dynamic pressure or airspeed, can be selected to match units desired by the pilot or engineer. The main point is this tool provides a quantitative value of the flight conditions at which flutter occurs. This tool can drastically increase flight test efficiency since test points can be safely chosen with greater rate of expansion of the flight envelope.

Motivation

- Analytical Predictions
  - *p-k* from FEM and CFD
  - 1st order perturbations
  - stochastic robustness



- On-Line Estimates
  - damping
  - parameter identification
  - modal filter

well-developed model ◄— **STRENGTHS** —► data describes true aircraft

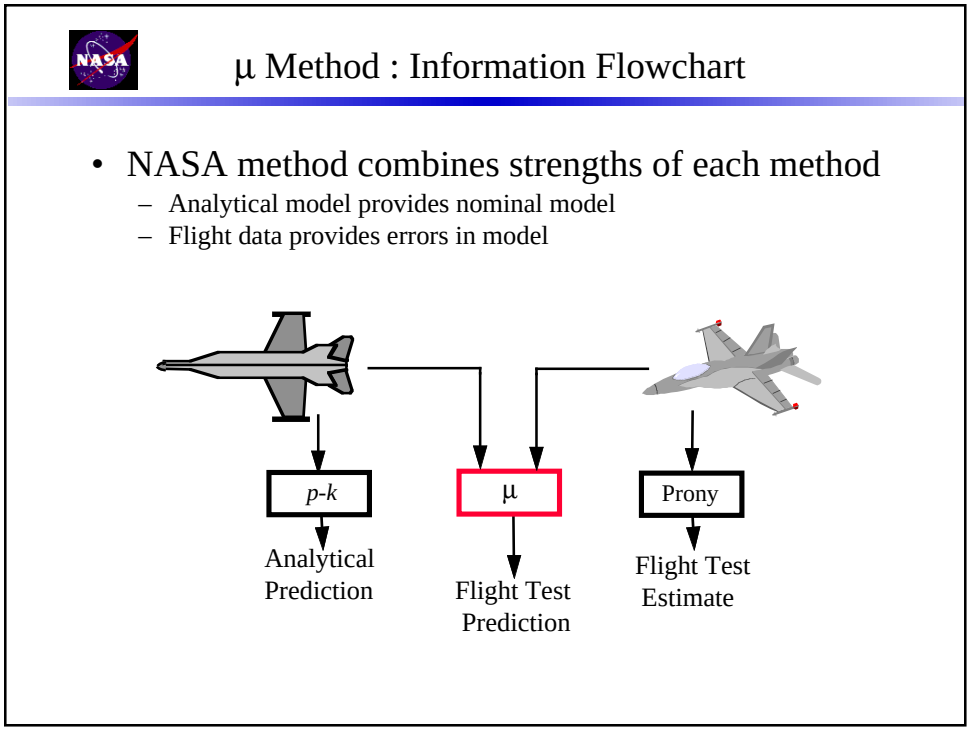
model is approximation ◄— **DRAWBACKS** —► can't extrapolate to margin

This flutterometer concept can not be effectively implemented using any current flutter analysis method. One explanation for this is seen by dividing all current methods into two basic categories called Analytical Prediction methods and On-Line Estimation methods. The Analytical Prediction methods, of which the *p-k* method is the most common, utilize a computational model with no direct consideration of flight data. The On-Line Estimation methods, of which tracking damping estimates is the most common, utilize the flight data alone. Each of these methods has strengths and weaknesses that are directly related. The drawbacks to the Analytical Prediction methods are eliminated by the On-Line Estimate methods; however, the On-Line Estimate methods introduce their own drawbacks which are not problematic for Analytical Prediction methods.

Limitations in Proposed Approaches

- Analytical Prediction Methods
  - 1st order perturbation analysis (Becus,Poiron)
    - perturbation structure may not be realistic
    - flutter margins may be overly conservative
  - Stochastic robustness (Stengel)
    - expensive Monte Carlo simulations
    - robustness levels are statistical with no guarantees
- On-Line Estimation Methods
  - Parameter estimation (Nissim,Feron)
    - computationally expensive
    - no convergence or optimality guarantees
    - poor performance for low SNR data
  - Modal filtering (Shelley,Allemang)
    - model based method is not adaptive or robust
    - possible problems for dense modal spaces
    - its a filtering, not a processing, algorithm

The most common methods of flutter analysis, namely the analytical *p-k* method and the on-line estimation of damping, have been recognized as deficient for many years. Several new methods are being investigated by various researchers to replace these traditional methods. These new methods still fall into the two basic categories and thus have the same strengths and drawbacks. These are additional limitations that should also be considered for these new methods.



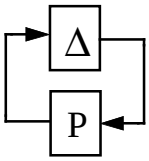
NASA Dryden Flight Research Center is developing a novel method for flutter analysis that combines the strengths of both categories of traditional flutter analysis. This method is essentially model based so it has the desired predictive nature of an Analytical Prediction method; however, this method, unlike traditional Analytical Prediction methods, can also utilize the flight data to obtain the desired accuracy of the On-Line Estimate methods. This method, referred to as the μ method, introduces a new category of analysis called Flight Test Prediction methods.

μ Method : Robust Stability

- μ uses mathematical representations
  - $\mathcal{H}_\infty$ operators : stable linear time-invariant transfer functions
  - $\mathcal{L}_2$ signals : finite bounded square-integrable measurements

- μ uses Linear Fractional Transformation framework
  - series and feedback interconnections of operators
  - multiple LFT's with unstructured operators
    result in single LFT with structured operators

$$\mathcal{P} = \left\{ F_u(P,\Delta) \ : \ \|\Delta\| \le 1 \right\}$$



The μ method is able to make strong claims about robustness by utilizing a well developed mathematical framework. The underlying concepts are derived from functional analysis and introduce abstractions such as $\mathcal{H}_\infty$ operators and $\mathcal{L}_2$ signals. These abstractions are readily interpreted as standard systems concepts such as transfer functions and data measurements. The Linear Fractional Transformation is used to represent series and feedback operators in a single unified framework. The concept of a set of plants, denoted $F_u(P,\Delta)$, will be used extensively throughout this presentation and refers to the block diagram in the Figure.

μ Method : Robust Stability

- Model should be robustly stable to uncertainty Δ
  - Parametric uncertainty describes errors in specific elements
    - Consider plant with uncertain pole

$$\mathcal{P} = \left\{ \frac{1}{(s+1)(s+x)} : x \in [2\ 3] \right\}$$

    - Express this plant in LFT form with Δ

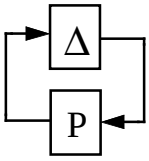
$$\mathcal{P} = \left\{ F_u(P,\Delta): \quad P = \begin{bmatrix} \frac{-1}{s+2.5} & \frac{-1}{s+2.5} \\ \frac{1}{(s+1)(s+2.5)} & \frac{1}{(s+1)(s+2.5)} \end{bmatrix}, \|\Delta\| \le 1, \Delta \in \mathcal{R} \right\}$$

  - Dynamic uncertainty is a more general type of variation
    - Multiplicative Uncertainty $\mathcal{P} = \{P(I+\Delta): \quad \|\Delta\|_\infty \le 1\}$
    - Additive Uncertainty $\mathcal{P} = \{P + \Delta \ : \ \|\Delta\|_\infty \le 1\}$

The concept of robustness is frequently used in engineering terminology to loosely refer to stability and performance of a system despite some concept of perturbations. The μ framework utilizes a formal definition of robustness and associated perturbations. The perturbations are represented by a set of norm bounded operators, Δ, which affect the plant, P, through a feedback relationship using the LFT framework. This uncertainty can be parametric uncertainty which affect specific elements of a system or general dynamic uncertainty which affects groups of signals.

μ Method : Robust Stability

- What does robust stability of P mean?
  - No perturbation Δ with ||Δ||<1 can destabilize P
  - Stability margin of P is greater than the size of Δ
  - Conservative condition for robust stability is ||P|| < 1



- Define the structured singular value μ

$$\mu(P) = \frac{1}{\min\limits_{\Delta \in \mathbf{\Delta}} \left\{ \bar{\sigma}(\Delta) \ : \ \det(I - P\Delta) = 0 \right\}}$$

  - multivariable gain/phase margin
  - relates smallest destabilizing perturbation

- μ is an exact measure of robust stability
  - uncertainty is weighted so μ=1 is the desired condition
  - upper bound computed via convex optimization

The formalized concept of robustness ensures the plant will be stable for any perturbation operator Δ contained within the set **Δ**. This concept is directly related to the concept of a stability margin. The structured singular value, μ, is defined as a necessary and sufficient condition to exactly compute the robustness of a plant operator. μ can be interpreted as a multivariable gain and phase margin but perhaps the most straightforward interpretation is that of the smallest destabilizing perturbation. The plant is usually weighted such that the set of uncertainty operators has a unity norm bound and the desired robustness condition is μ=1.

μ Method : Robust Stability

- Model validation considers data and model
  - Compares expected measurements with actual measurements
  - Provides measure of plant accuracy
  - Actually tests if model is not invalidated
    since model can not be truly validated by a finite amount of data

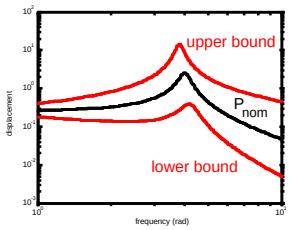
Question : Is there a Δ in the set $\mathbf{\Delta}$ such that $F_u(P,\Delta)$
could generate measured *y* in response to *u*

- μ can be used as a model validation criterion
  - Form matrix $\underline{P} = f(P,u,y,\omega)$
  - The model is not invalidated if $\mu(\underline{P})>1$

A difficulty in using μ to compute robust stability lies in choosing a set of meaningful uncertainty operators. Model validation algorithms are formulated to address this issue by comparing measured data to the set of plants generated by $F_u(P,\Delta)$. The model validation criterion determines there exists some Δ in the set $\mathbf{\Delta}$ such that the plant $F_u(P,\Delta)$ could have generated the measured data. This criterion actually ensures the model is not invalidated since a model can never by truly validated by a finite set of data; however, even this restricted condition is useful to ensure the uncertainty set is reasonable.

μ Method : Robust Stability

- μ model validation is transfer function based
  - experimental data must fall between upper and lower bounds
  - frequency-varying bounds are determined by uncertainty
  - damping is not estimated



- μ is used to consider all Δ blocks simultaneously

A approach to model validation in the μ framework is developed using frequency domain transfer function data. The basic concept considers upper and lower bounds for the set of plant models with uncertainty and determines if the measured transfer function lies within those bounds. This approach uses a simple test at each frequency point without requiring estimates of specific system and modal parameters.

μ Method : Flutter Applications

- μ has well-known interpretations for controls
  - Gain and phase margins
  - Closed-loop performance despite actuator and sensor noise
- μ has interpretations also for aeroelasticity
  - Describes traditional flutter margin similar to *p-k* analysis
  - Describes robust flutter margin for plants with modeling errors
- μ Method is valuable tool for flutter analysis
  - Method incorporates flight data
  - Margins are worst-case with respect to some uncertainty
  - μ replaces poorly behaved damping as a stability margin

μ was originally developed for control design and analysis and is a well-known concept in that field of engineering. NASA Dryden Flight Research Center is developing applications of μ for aeroelasticity research and has found several interpretations for this field. μ can be formulated to compute a flutter margin which is similar to that generated by traditional analysis methods such as the *p-k* method. μ can also represent a robust flutter margin that considers the worst-case flutter margin with respect to modeling errors and perturbations represented by the set $\mathbf{\Delta}$.

μ can be especially valuable for flutter analysis. The confidence in the computed flutter margins are high due to the inclusion of flight data which describes the true aircraft. The formalized derivation of μ lends some mathematical guarantees that the computed robust margins are worst-case with respect to a set of perturbations. Also, μ is continuous and smooth with flight condition variation so replaces the poorly behaved nonlinear damping trend parameter to track during a flight test.

μ Method : Nominal Flutter Margins

- Consider computing traditional flutter margins
  - traditional margins only use the theoretical plant
  - traditional margins do not consider uncertainty (nominal)

Question : How can μ represent a traditional flutter margin?

- μ relates destabilizing perturbation
- Flutter margin relates unstable flight condition

Answer : μ is destabilizing perturbation to flight condition

The first step in developing the μ method is to generate flutter margins which are equivalent to traditional margins generated by the *p-k* method. These margins do not account for any uncertainty and are denoted as nominal margins in the μ vernacular. μ is formulated as a flutter margin by considering the smallest destabilizing perturbation to a flight condition.

μ Method : Nominal Flutter Margins

Consider equation of motion for state vector η

$$M\ddot{\eta} + C\dot{\eta} + K\eta + \bar{q}Q(s)\eta = 0$$

Structural Dynamics
- M - mass matrix
- C - damping matrix
- K - stiffness matrix

Unsteady Aerodynamics
- Q - forces matrix
- q - dynamic pressure

Represent unsteady aerodynamics as transfer function

$$Q(s) = \begin{bmatrix} A_Q & B_Q \\ C_Q & D_Q \end{bmatrix} = D_Q + C_Q(sI-A_Q)^{-1}B_Q$$

The equations of motions for an aeroelastic system utilize a structural model and an unsteady aerodynamic force model. The unsteady aerodynamic forces can be represented as a finite-dimensional state-space model using several standard algorithms.

μ Method : Nominal Flutter Margins

Consider parametric perturbation in dynamic pressure

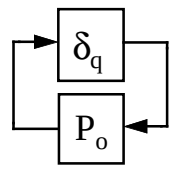
$$\bar{q} = \bar{q}_o + \delta_{\bar{q}}$$

Separate perturbation from nominal dynamics

$$\begin{aligned}
0 &= M\ddot{\eta} + C\dot{\eta} + K\eta + \bar{q}Q\eta \\
&= M\ddot{\eta} + C\dot{\eta} + K\eta + \left(\bar{q}_o + \delta_{\bar{q}}\right)\left(C_Q x + D_Q \eta\right) \\
&= M\ddot{\eta} + C\dot{\eta} + K\eta + \bar{q}_o\left(C_Q x + D_Q \eta\right) + \delta_{\bar{q}}\left(C_Q x + D_Q \eta\right) \\
&= M\ddot{\eta} + C\dot{\eta} + K\eta + \bar{q}_o\left(C_Q x + D_Q \eta\right) + \delta_{\bar{q}} z \\
&= M\ddot{\eta} + C\dot{\eta} + K\eta + \bar{q}_o\left(C_Q x + D_Q \eta\right) + w
\end{aligned}$$

Formulate LFT for μ analysis as $F_u(P_o,\delta_q)$

$$\begin{bmatrix} \dot{\eta} \\ \ddot{\eta} \\ \dot{x} \\ z \end{bmatrix} = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ -M^{-1}(K+\bar{q}_o D_Q) & -M^{-1}C & -M^{-1}\bar{q}_o C_Q & -M^{-1} \\ B_Q & 0 & A_Q & 0 \\ \hline D_Q & 0 & C_Q & 0 \end{array}\right] \begin{bmatrix} \eta \\ \dot{\eta} \\ x \\ w \end{bmatrix}$$



The dynamic pressure effects the equations of motion in a linear manner so perturbations to this flight condition parameter may be easily represented with a feedback operator. The basic procedure is to isolate the perturbation from the known nominal dynamics and replace this perturbation with a norm bounded operator.

μ Method : Nominal Flutter Margins

- μ directly computes nominal flutter margins
  - range of dynamic pressures is treated as uncertainty δ_q
  - μ considers stability over all uncertainty (dynamic pressures)

Nominal Flutter Question :

- What is the smallest $\bar{q}$ for which $P(\bar{q})$ is unstable? (*p-k*)
- What is the smallest δ_q for which $F_u(P_o,\delta_q)$ is unstable? (μ)

- Nominal μ and *p-k* margins should be similar
  - both methods use the same theoretical plant model
  - both methods use the model with no accounting for uncertainty

μ computes a flutter margin for the nominal system by considering the smallest perturbation to dynamic pressure which causes an instability. Considering all perturbations to dynamic pressure allows μ to extrapolate to the flutter boundary from a particular stable flight condition. The nominal margins computed with the μ and *p-k* methods should be similar since both methods utilize the same theoretical aeroelastic model with no accounting for modeling errors.

μ Method : Robust Flutter Margins

- Uncertainty should be included in model
  - Nominal model is not accurate
  - Flutter margins should consider errors in the model
- Formulate uncertain P_o at nominal $\bar{q}_o$
  - Add feedback operator δ_q to account for flight conditions
  - Add feedback operator Δ to account for modeling uncertainty
- μ computes robust flutter margins
  - Utilize block diagonal uncertainty $\hat{\Delta}=\{\delta_q,\Delta\}$
  - Iterate over scalings on δ_q until $\mu=1$
  - μ is worst-case δ_q with respect to Δ

The μ method truly becomes a unique valuable tool for flutter analysis by accounting for modeling uncertainty. This uncertainty is essential since the nominal model is never exactly accurate so the flutter margins computed with that nominal model may be arbitrarily different than the margins of the true aircraft. The basic procedure for including uncertainty is to introduce feedback operators to appropriate elements of the system and utilize the LFT framework to express an uncertain plant model comprised of a nominal plant model and a single structured uncertainty set. A process iterating between adjustments to norm bounds on the uncertainty and computations of μ can be used to compute a flutter margin associated with the desired $\mu=1$ condition.

μ Method : Robust Flutter Margins

Consider Pade approximation of Q with lag uncertainty

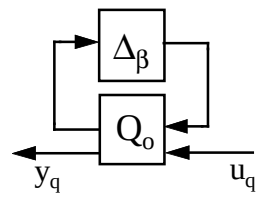
$$y_q = Q(s)u_q = \frac{s}{s+\beta_o(1+\Delta\beta)}\,u_q$$

Separate perturbation from nominal state equation

$$\begin{aligned}
\dot{x} &= -\beta x - \sqrt{\beta}u_q \\
&= -\beta_o(1+\Delta\beta)x - \sqrt{\beta_o}(1+\Delta\beta)u_q \\
&= -\beta_o x - \sqrt{\beta_o}u_q + \Delta\beta\left(-\beta_o x - \sqrt{\beta_o}u_q\right) \\
&= -\beta_o x - \sqrt{\beta_o}u_q + \Delta\beta z \\
&= -\beta_o x - \sqrt{\beta_o}u_q + w
\end{aligned}$$

Formulate LFT as $F_u(Q_o,\Delta_\beta)$

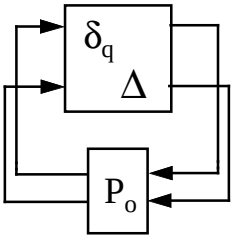
$$\begin{bmatrix} \dot{x} \\ z \\ y_q \end{bmatrix} = \left[\begin{array}{c|cc} -\beta_o & 1 & -\sqrt{\beta_o} \\ \hline -\beta_o & 0 & -\sqrt{\beta_o} \\ \sqrt{\beta_o} & 0 & 1 \end{array}\right] \begin{bmatrix} x \\ w \\ u_q \end{bmatrix}$$



An example of modeling uncertainty arises when considering aerodynamic lag from a Pade approximation. These lag terms may have slight errors so a perturbation can be introduced to account for the errors. The perturbation can be separated from the nominal dynamics and extracted as an uncertainty operator introduced in a feedback manner.

μ Method : Robust Flutter Margins

- Denote uncertainty $\hat{\Delta}=\mathrm{diag}\{\delta_q,\Delta\}$
  - Parameterization of Dynamic Pressure
  - Structured Uncertainty



Nominal Flutter Question :

- What is the smallest δ_q for which $F_u(P_o,\delta_q)$ is unstable?

Robust Flutter Question :

- What is the smallest δ_q for which $F_u(P_o,\hat{\Delta})$ is unstable for some Δ in Δ?

All the individual uncertainty operators, such as the lag uncertainty, are combined into a single structured uncertainty operator using the LFT framework. The nominal flutter margins are computed by considering the smallest destabilizing perturbation to dynamic pressure that causes an instability in the nominal plant P. The robust flutter margins are computed by considering the smallest destabilizing perturbation to dynamic pressure that causes an instability in any member of the set of plants $F_u(P,\Delta)$.

μ Method : Robust Flutter Margins

- What does a robust flutter margin really mean?
  - $F_u(P,\Delta)$ is a set of plant models including nominal and perturbed dynamics
  - Each perturbed plant in the set has a flutter margin
  - Robust flutter margin is the smallest margin of all margins for the set of plants
- Robust flutter margins are conservative
  - Uncertainty can only decrease stability margin
  - Robust μ margin is never greater than nominal μ margin

The robust flutter margin concept seems a significant departure from traditional flutter margins; however, the robust flutter margin can be interpreted in terms of nominal flutter margins. A unique plant model is obtained for each uncertainty operator in the set and a nominal flutter margin can be computed for each of these unique plant models. The robust flutter margin is simply the smallest margin of these individual margins. The operator theory and functional analysis concepts are introduced to ensure the robust flutter margin analysis considers all uncertainty operators in the set.

μ Method : Incorporating Flight Data

- Flight data should be utilized in the analysis
  - Theoretical P may not be accurate
  - Theoretical Δ may not be reasonable
- Flight data can be incorporated in several ways
  - Formulate $F_u(P,\Delta)$ (difficult)
  - Formulate P (problematic)
  - Formulate Δ (advantageous)
- An approach
  - Utilize the original theoretical plant as the nominal model
  - Choose location and structure for uncertainties
  - Use model validation to choose size of uncertainties

Flight data provides the only true indication of the aircraft properties and should be included in the flutter margin analysis. The optimal approach would use this data to directly identify parameters in the model and an associated uncertainty set but this is extremely difficult. Approaches to utilize the data to identify a plant model are valuable but problematic since they are unreliable with poor quality data. The approach investigated at NASA Dryden Flight Research Center is to utilize the flight data to identify the errors and uncertainty in a theoretical plant model.

μ Method : Incorporating Flight Data

- Why formulate only Δ with flight data?
  - Computational algorithm is straightforward
    - Formulating P via nonlinear optimization is nonunique
    - Δ can be updated with simple scalar multiplication
  - Method works with poor quality flight data
    - Use *a priori* theoretical Δ_o if data can't be used for validation
    - Δ_o can be used for all or specific frequencies
  - Method accounts for time-varying behavior
    - Difficult to choose best data to formulate P
    - Δ can be norm bound accounting for all time variations
  - Exact model parameters do not have to be computed
    - Parameter estimation of P is difficult with poor data
    - Model validation is transfer function based

There are several advantages to using flight data to compute the uncertainty description as compared to identifying plant model parameters. One main advantage is the ability to utilize data with varying levels of noise. Another advantage, which will be utilized for the on-line implementation, is the ability to account for time-varying dynamics of the airplane.

μ Method : Incorporating Flight Data

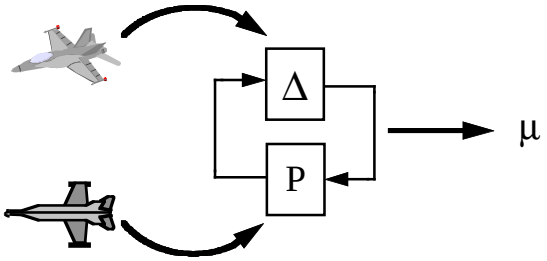
Flight data can validate the uncertainty
so Δ is a reasonable indicator of modeling errors

- Choose Δ large enough to be accurate
  - Increase size of Δ so no data can invalidate the model
  - Increase size of Δ at particular frequencies where data differs from P
  - Increase size of particular blocks of Δ
- Choose Δ small enough to reduce conservatism
  - Reduce size of Δ so some data almost invalidates the model
  - Reduce size of Δ at particular frequencies
  - Reduce size of particular blocks of Δ

An uncertainty description must be chosen that is a reasonable indication of modeling errors. This implies a tradeoff must be met between choosing Δ large enough to account for all errors but choosing Δ small enough to reduce conservatism in the robust margins.

μ Method : Incorporating Flight Data

- Update robust model through Δ
  - Finite element model does not change
  - Δ may be determined from flight data observations



- μ flutter margins are worst-case
  - μ is worst-case with respect to uncertainty
  - μ is worst-case with respect to errors observed with flight data

The μ method can be formulated using a theoretical plant model and measured flight data. The theoretical plant model is assumed to be the best estimate of the aircraft dynamics so this model is not changed throughout the analysis. The flight data is incorporated entirely through the uncertainty operators. μ computes a flutter margin for the theoretical plant that is worst-case with respect to the uncertainty and thus is worst-case with respect to the observed flight data.

μ Method : Robust Flutter Margins

Include uncertainties to formulate robust model.

- Uncertainty from Modeling Principles
  - Unmodeled dynamics and nonlinearities
  - Errors in structural elements and parameters
  - Inaccuracies in unsteady dynamic force model
- Uncertainty from Flight Test Data
  - Measurement of excitation
  - Nonrepeatibility
- Uncertainty from Signal Processing
  - Waveform basis
  - Assumptions of stationarity, linearity, time-invariance
- Uncertainty from control surfaces on aircraft
  - Nonlinear actuators
  - Hysteresis and Freeplay

There are many sources of uncertainty in a plant model. The obvious sources are errors in the theoretical plant dynamics arising from inaccuracies in the structural and aerodynamic models. Utilizing flight data to validate the model introduces additional uncertainty since the data measurement and processing may not be accurate. Also, the aircraft may display behaviors for which the measured excitation signal does not account.

μ Method : Properties of μ

μ has several desirable features as a stability margin as compared to parameters such as damping

- Conservatism is a measure of sensitivity
  - Models sensitive to errors will be conservative to uncertainty
- μ analysis can determine worst-case perturbation
  - Indicates worst-case flutter mechanism
  - May indicate active and passive flutter control strategies
- μ is a stability predictor
  - Damping is only guaranteed informative at instability
  - μ extrapolates across flight condition
  - μ is linear across dynamic pressures with no Δ
  - μ is generally well-behaved across dynamic pressure with Δ

There are several desirable properties of μ that make this parameter an advantageous flutter margin as compared to damping. μ presents more information than damping since the conservatism between the robust and nominal margins is an indication of modeling sensitivity. Most importantly, μ is a stability predictor and extrapolates to an unstable flight condition whereas damping is merely a stability indicator and is only truly informative at the point of instability.

μ Method : Assessment

- Limitations
  - Model based
  - Linear
- Difficulties
  - Difficult to choose structure of uncertainty
  - Difficult to choose method of updating the model
  - Difficult to choose flight data used in model validation

The μ method is a powerful tool for flutter margin analysis but it is only useful when its limitations and difficulties are understood. This method is limited to applications of analyzing stability of aircraft with a plant model comprised of linear operators. The main difficulty lies in choosing the uncertainty description. μ is directly related to this uncertainty description so the robust flutter margin may be meaningless if the uncertainty description is meaningless.

μ Method : Assessment

- Advantages
  - Can utilize good and bad quality flight data
  - Can use off-the-shelf hardware and software
  - Computational cost is often not extravagant
  - Applied to a real problem and got acceptable results
- Several situations where μ method is ideal
  - Want to consider several variations of a model (configurations)
  - Know locations and bounds on model element variations
  - Want some indication of model sensitivity
  - Modeling errors are not huge
  - Want closed-loop margins with a controller (ASE)

The μ method, despite its limitations and difficulties, can be effectively applied to many aircraft applications. This method is ideal for considering several variations and configurations of an aircraft that only slightly perturb the nominal model. This method is also ideal for analyzing aeroservoelastic stability since it is a trivial extension to include a controller in the formulation and consider the closed-loop dynamics.

Worst-Case Flutter Margins : Post-Flight Analysis

- μ Method can be utilized for post-flight analysis
  - Aircraft demonstrated certain stability properties at test points
  - Flight data should be used to update the model
  - μ computes worst-case flutter margins with respect to Δ
- Post-flight analysis with μ can be valuable
  - Determine worst-case flight envelope
  - Identify inaccuracies and sensitivities in a model
  - Demonstrate test points that should be repeated

The μ method can be utilized for post flight data analysis to compute worst-case flutter margins and determine a worst-case flight envelope. The conservatism between the robust and nominal margins demonstrates the sensitivity of the model and may indicates areas of the flight envelope for which more data should be generated due to dramatic differences between the theoretical nominal and flight derived robust margins.

Worst-Case Flutter Margins : Post-Flight Analysis

- Available information
  - Nominal aeroelastic model P(M) at different Mach numbers
  - Multiple data sets
    - Multiple test points at identical flight conditions
    - Multiple test points at varying flight conditions
- Choose uncertainty Δ from data at certain Mach
  - $\Delta(M)$ varies with Mach number (local)
  - Δ constant for all Mach numbers (global)
  - $\Delta(M)$ is piecewise-constant (hybrid)
- Approach : compute $\mu\left(F_u(P(M),\Delta(M))\right)$ for each M
  - Size of $\Delta(M)$ relates model accuracy
  - Conservatism of μ relates model sensitivity
  - Value of μ relates worst-case flutter margin

There are several approaches that may be used for deriving an uncertainty description from multiple sets of flight data. This uncertainty description is formulated as a function of Mach number to associate with each plant model describing the dynamics at each Mach number. A robust flutter margin is computed for the associated plant and uncertainty models at each of these Mach numbers and the worst-case flight envelope is computed with respect to both dynamic pressure and Mach flight conditions.

Worst-Case Flutter Margins : Δ

- Local approach uses unique Δ(M) at each Mach
  - Model validation only considers data sets from that Mach
  - Advantage : less conservative
  - Disadvantage : susceptible to poor data at a particular Mach
- Global approach uses constant Δ at all Mach
  - Model validation considers data sets from all Mach numbers
  - Advantage : Δ is worst-case with respect to all test points
  - Disadvantage : conservative at well-modeled test points
- Hybrid approach uses piecewise-constant Δ(M)
  - Model validations considers all data from certain Mach numbers
  - Advantage : Δ is reasonable for conservatism and accuracy
  - Disadvantage : may be difficult to group Mach numbers

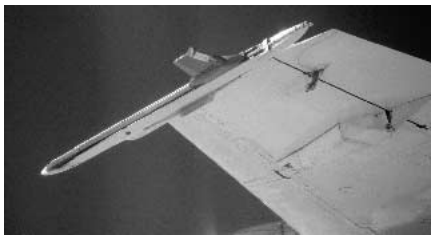
The choice of which data sets to analyze affects the uncertainty description. A local approach generates an uncertainty description for the plant at a particular Mach number by only considering flight data measured at that Mach number. A global approach considers all sets of flight data and generates a single uncertainty description that is worst-case with respect to variations observed throughout the flight envelope. A hybrid approach can be used to group sets of flight data together by assuming certain Mach numbers at certain test points, such as subsonic flight conditions, are strongly related to each other but not to other flight regions.

Worst-Case Flutter Margins : F/A-18 SRA

- F/A-18 Systems Research Aircraft
  - well developed finite element model
  - complete set of *p-k* flutter solutions

Post flight analysis is used to compute worst-case flutter margins for the F/A-18 Systems Research Aircraft (SRA). This aircraft is an excellent application for this method since a well-developed theoretical model is available along with a complete set of nominal *p-k* flutter margins.

Worst-Case Flutter Margins : F/A-18 SRA



- Flight data from Wingtip Exciters
  - multiple sine sweeps from 3 to 35 Hz
  - symmetric and antisymmetric operation
- Test points throughout flight envelope
  - Mach range : $0.6 < M < 1.4$
  - altitude range : $10 \text{ kft} < h < 40 \text{ kft}$

The F/A-18 SRA is also an excellent application since a large database of flight data has been recorded in response to a set of wingtip exciters. The test points for these response measurements cover a large range of flight conditions throughout the flight envelope.

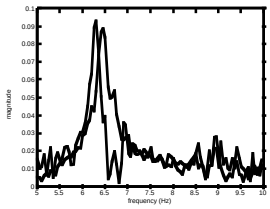
Worst-Case Flutter Margins : F/A-18 SRA

- State-space model aeroelastic dynamics
  - Finite element structural model has 34 modes
    - 6 rigid body modes
    - 14 symmetric elastic modes
    - 14 antisymmetric elastic modes
  - Unsteady aerodynamic model has 84 states
    - 56 states for symmetric forces
    - 28 states for antisymmetric forces
- Input and Output signals on each side of the aircraft
  - 5 output accelerometers
  - 1 input exciter force

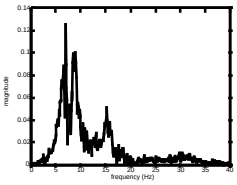
The state-space model for this system includes 34 modes from the structural model and 84 states accounting for the unsteady aerodynamic forces. There are 5 accelerometers measurements on each wingtip of the aircraft with a measurement of excitation force on each wingtip.

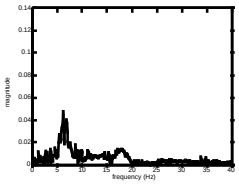
Worst-Case Flutter Margins : F/A-18 SRA

- Uncertainty in Modeling
  - Parametric
    - Mass
    - Damping
    - Aerodynamics
  - Dynamic
    - Nonlinearities
    - Unmodeled dynamics



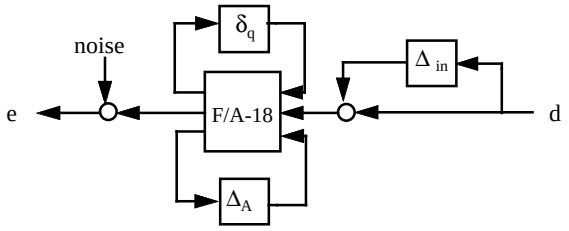
- Uncertainty in Flight Data
  - Flight Test
    - Exciter Performance
    - Nonrepeatibility
    - Flight Path
  - Fourier Processing
    - Time-varying transients
    - Nonlinear behavior





Areas of modeling uncertainty can be seen by visually inspecting some flight data sets. The top plot demonstrates two flight data transfer functions taken at identical flight conditions. These transfer functions show a small variation of approximately .4 Hz in the modal natural frequency of the Wing 1st Bending mode. The bottom plots demonstrate the concept of nonrepeatibility that affects flight flutter testing. These two plots show transfer functions generated by response data at identical flight conditions for which the modal frequencies and levels displayed are clearly different.

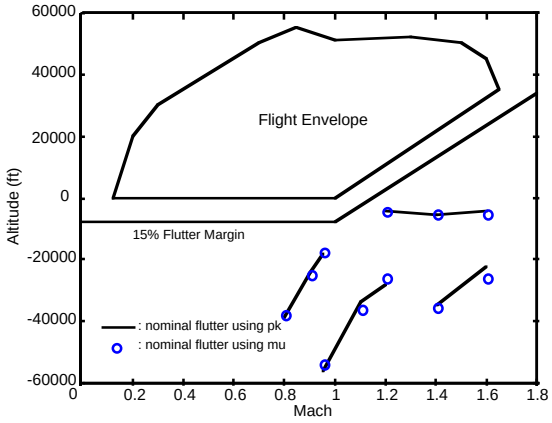
Worst-Case Flutter Margins : F/A-18 SRA



- Δ_A : modal uncertainty
  - 5% in natural frequency
  - 15% in damping
  - 15% in aerodynamic lags
- Δ_{in} : input uncertainty
  - 10% at low frequency
  - 100% at 150 Hz
  - 500% at high frequency

An uncertainty description is chosen for the F/A-18 SRA based on the observed flight data variations. A parametric modal uncertainty operator is chosen to introduce 5% uncertainty in natural frequencies, 15% uncertainty in damping ratios and 15% uncertainty in aerodynamic lags. A complex dynamic input multiplicative uncertainty is included to introduce 10% uncertainty at low frequencies and increasing to demonstrate the model is poor at high frequencies. Sensor noise is included along with the perturbation to dynamic pressure that allows μ to extrapolate to the flutter margin.

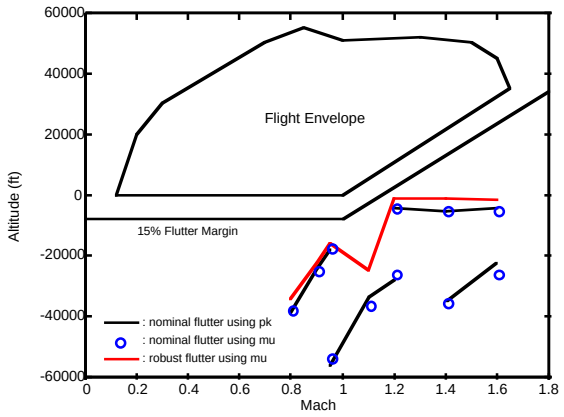
Worst-Case Flutter Margins : F/A-18 SRA



Nominal Symmetric Flutter Points

The initial calculation of flutter margins considers only the nominal plant dynamics with no consideration of the uncertainty description. This plot shows the *p-k* margins as solid lines and the nominal μ margins as circles. These values are quite similar to indicate the μ method can accurately compute flutter margins. The frequencies of each unstable flutter mode are not given on this plot but are shown in the published reference documents to match closely.

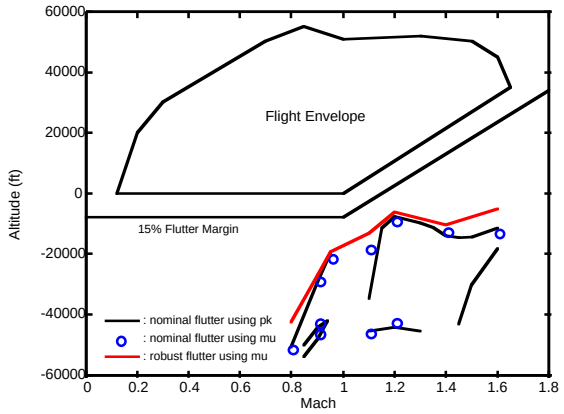
Worst-Case Flutter Margins : F/A-18 SRA



Symmetric Flutter Margins

Robust flutter margins are computed for the symmetric modes using μ with respect to the uncertainty description.
The dashed line indicates these worst-case margins which are more conservative and closer to the flight envelope than the nominal margins. The robust flutter margins at transonic flight conditions are particularly conservative.

Worst-Case Flutter Margins : F/A-18 SRA

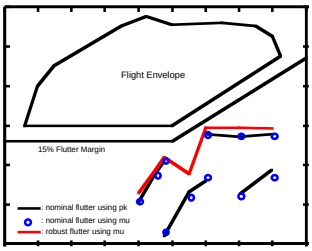


Antisymmetric Flutter Margins

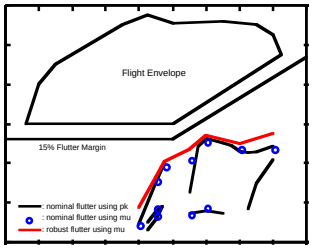
Robust flutter margins are also computed for the antisymmetric modes and are shown by the dashed line in this plot. These margins show a similarity to the symmetric mode margins in that the transonic flight condition is especially sensitive to modeling uncertainty.

Worst-Case Flutter Margins : F/A-18 SRA

- Features of Symmetric
  - Similar margins at supersonic
  - Worst-Case at M=1.2 close to 15% margin



- Features of Antisymmetric
  - High sensitivity at M=1.6
  - Subcritical hump mode at M=.95



- Features of Both
  - Poor nominal match at M=1.1
  - High sensitivity at M=1.1
  - Worst-case margins outside 15% margin

The entire set of robust flutter margins can be considered by evaluating the margins for the symmetric and antisymmetric modes. These margins, despite the conservatism associated with including uncertainty, are more than 15% in airspeed from the flight envelope of the F/A-18 SRA.

Flutterometer : On-Line Implementation

- Extend method to on-line predictions during flight
  - Update uncertainty description at each test point
  - Update flutter margin at each test point
- Flutterometer displays on-line predictions
  - μ prediction directly accounts for flight data
  - μ prediction extrapolates to flutter boundary
- Flutterometer can improve flight test efficiency
  - Test point data tracks time-varying dynamics
  - Worst-case margins provide confidence
  - μ is a better behaved stability margin as compared to damping
  - Confident margins can be used to adapt flight plan

The μ method for flutter analysis is extended to an on-line implementation by considering a test point approach. Flight data is gathered at each test point and immediately used to generate an uncertainty description and an associated robust flutter margin. The robust flutter margin information at each test point is displayed via the flutterometer tool which indicates the distance to flutter for that test point. This approach allows the flutterometer to track time-varying dynamics of the airplane since the uncertainty description is continually updated as flight data is measured and the μ margin accounts for the time-varying dynamics through this time-varying uncertainty.

Flutterometer : On-Line Implementation

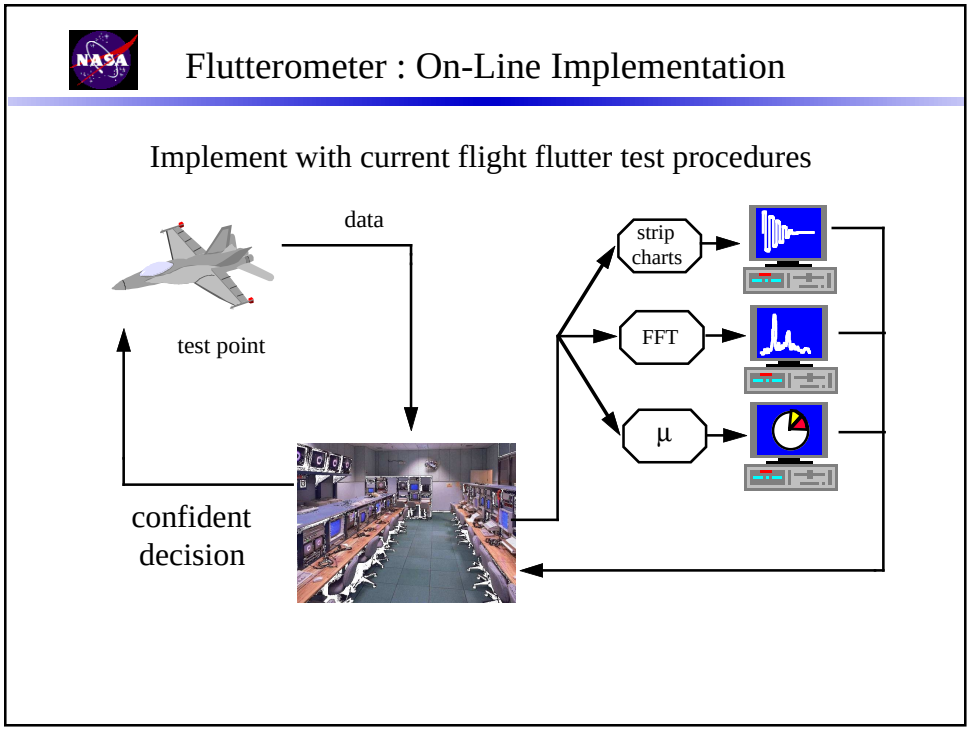
- Several methods to choose Δ during a flight test
  - Δ considers data from current test point (local)
  - Δ considers data from all test points (global)
  - Δ considers data from recent test points (hybrid)
- Approach is similar to current methods
  - Trim at a stabilized test point
  - Record transfer function data
  - Transfer data to analysis computer
  - Estimate modal damping
    Determine reasonable uncertainty Δ
    Compute worst-case flutter margin μ
  - Determine conditions for next, if any, test point

The μ method can be implemented in a manner similar to computing damping estimates. The flight data can be simultaneously analyzed by traditional methods and the μ method on separate computers. The flight conditions for the next test point can be determined using the combination of information from the traditional damping estimate and the new μ method.

Flutterometer : On-Line Implementation

- Local approach uses current data
  - Only data from current test point is used
  - Advantage : less conservative
  - Disadvantage : susceptible to poor data set
- Global approach uses all data from the flight
  - Model validation considers data from all previous test points
  - Advantage : worst-case with respect to range of flight conditions
  - Disadvantage : may be overly conservative
- Hybrid approach uses recent flight data
  - Model validation considers current and recent data
  - Advantage : reasonable for conservatism and accuracy
  - Disadvantage : may be difficult to define recent

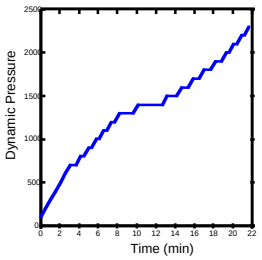
The choice of flight data to utilize at each test point allows flutter margins to be computed which are worst-case with respect to different uncertainty operators. A local approach uses only data from the current test point while a global approach uses all data from previous test points. A hybrid approach uses a forgetting factor and only uses data from recent test points.



The flutterometer based on the μ method can be easily implemented using current flight flutter test procedures. The only change is an additional analysis operation using the recorded flight data that computes a robust flutter margin. The resulting decision on future test points is considered confident since the damping estimates are accurate indicators of the current stability properties and the μ method provides significant additional information about the distance to a flutter instability.

Flutterometer : F/A-18 simulation

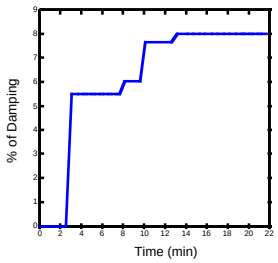
- Simulate flight flutter test of F/A-18 SRA
  - constant Mach dive at M=1.2
  - test points every 100 lb/ft^2 of q
- Test point procedure
  - Record frequency response data
  - generate uncertainty description
    - Validate current uncertainty levels
    - Increase if necessary
  - Compute worst-case μ flutter margin
- Time is variable in the simulation
  - computation time to validate Δ
  - computation time to compute μ



A simulated flight flutter test of the F/A-18 SRA demonstrates the flutterometer concept. The aircraft is undergoing a constant Mach dive at M=1.2 to expand the envelope. Test points are chosen at every 100 units of dynamic pressure to illustrate the on-line computations with some detail. The time spent at each of these test points is determined entirely by the amount of computation time required to compute an uncertainty description and an associated robust flutter margin with μ. This plot shows the value of dynamic pressure throughout the simulation with the length of the horizontal lines indicating the time spent at a test point for which the flight condition did not change while μ is being computed.

Flutterometer : F/A-18 simulation

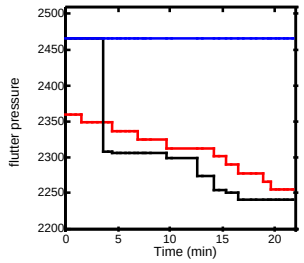
- Plant model is not accurate
  - error in structural damping (true aircraft is 10% higher)
  - error in initial mass value (true aircraft is 95% of heavyweight)
  - unmodeled time-varying mass (true aircraft varies 5% in 20 minutes
- Modal uncertainty increases as plant dynamics change
  - Modal parameters are time-varying
  - Δ increases as mass decreases



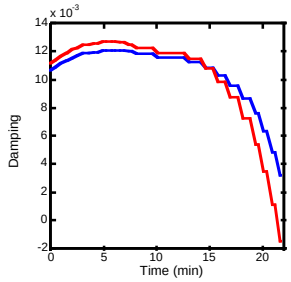
A time-varying theoretical plant model of the F/A-18 SRA is used as the true plant model while a variation of this model is used as the nominal dynamical model. The nominal model has an error in structural damping and does not account for the time-varying mass of the true aircraft. The plot shows the increase in modal damping uncertainty that is required throughout the simulation to ensure the flight data recorded at each test point does not invalidate the uncertain model.

Flutterometer : F/A-18 simulation

- Worst-Case On-Line Predictions
  - Track time-varying dynamics
  - Predict distance to flutter
  - Computation times of 1-3 minutes



- Traditional On-On-Line Estimates
  - Damping does not vary until minute 17
  - Trend does not extrapolate to flutter



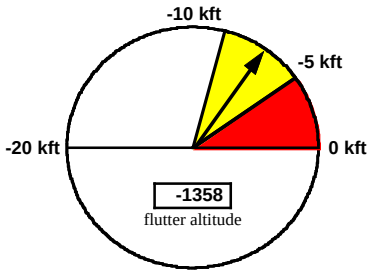
The top plot shows the true and computed flutter margins for each test point. The dashed line is the flutter margin of the true aircraft and decreases with time due to the time-varying mass of the aircraft. The solid line displays the robust flutter margin and also demonstrates a time-varying behavior due to the calculation of an uncertainty description and μ at each test point. This robust flutter margin utilizes uncertainty to account for the time-varying mass and remains conservative to the true flutter margin throughout the simulation. The dotted line near the top of the plot is the nominal flutter margin which does not account for any uncertainty in the nominal model.

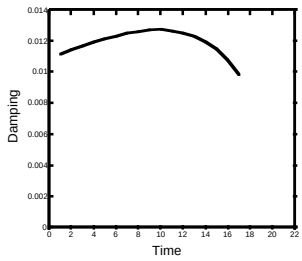
The bottom plot shows the modal damping ratio at each test point for the mode that goes unstable at the M=1.2 flight condition. The damping remains fairly constant until minute 17 when it decreases sharply to indicate the oncoming instability.

Flutterometer : F/A-18 simulation

Consider minute 17 test point :

- M = 1.2
- q = 1700 lb/ft^2
- h = 6147 ft



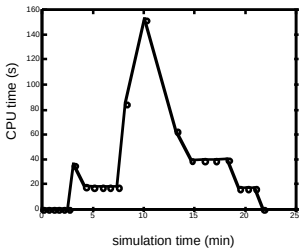


Damping trend indicates potential impending instability but flutterometer quantifies distance to instability

These plots demonstrate the information displayed in the control room at the minute 17 test point. The flutterometer on the left indicates the aircraft can drop 7500 feet before a flutter condition is encountered. The damping trend on the right indicates an instability may be near but the flight conditions associated with that instability can be not accurately determined from this trend. These plots clearly show the usefulness of a valid flutterometer tool for flight flutter testing.

Flutterometer : F/A-18 simulation

- Computations used standard equipment
  - Pentium 200 MHz computer
  - MATLAB and μ-Tools software
- CPU time was reasonable
  - time to validate Δ
  - time to compute μ



- What affects CPU time?
  - Number of uncertainties
  - Number of validation iterations
  - Number of frequency points to compute μ
  - Number of states in the model

This simulation uses standard hardware and software often available in a flight data analysis facility. The computational time required at each test point is reasonable and does not introduce an excessive burden on a flight test program. The plot uses a star symbol to indicate the CPU time required at any test was not greater than 180 seconds.

Flutterometer : Assessment

Flutterometer has same limitations/advantages as μ method with additional issues due to the implementation

- Limitations
  - Needs stabilized test points
  - Needs constant Mach envelope expansion
  - Needs transfer function data (output and input)
  - Needs some computation time (not real -time)
- Advantages
  - Tracks time-varying dynamics and flutter margins
  - Computational time is reasonable
  - μ is better behaved stability parameter than damping

The flutterometer tool is based on the μ method and has the same limitations and advantages as the μ method with several additional considerations. This tool is only directly applicable for flight programs using stabilized test points that can vary dynamic pressure while keeping Mach constant. The flutterometer can be extremely useful if these limitations are not problematic since the time-varying flutter margins can be tracked and μ is a much better behaved stability parameter than damping.

Conclusions

- μ method is improvement to traditional methods
  - Flight data and model are utilized
  - Worst-case flutter conditions are determined
- Flight data can be utilized in different ways
  - Post-flight analysis to update stability margins
  - In-flight analysis to update stability margins
- Flutterometer can improve flight test efficiency
  - Test point data tracks time-varying dynamics
  - Worst-case margins provide some level of confidence
  - μ replaces poorly behaved damping as a stability margin

The μ method is a significant improvement to traditional flutter margin analysis methods since it directly accounts for modeling uncertainty by including flight data. This method can be used in several ways including an on-line implementation to develop a flutterometer flight test tool. This flutterometer can improve flight test efficiency by providing a confident measure of the distance to flutter so test points can be chosen that quickly and safely expand the flight envelope.

μ Method : Research Extensions

- Aeroservoelastic stability margins
  - flutter and buffet stability margins
  - gain and phase controller margins for elastic plant
- Reducing conservatism in stability margins
  - Wavelet filtering to reduce conservatism
  - Model updating to reduce conservatism
- Analyzing nonlinear and LCO dynamics
  - Using μ to predict certain classes of transonic LCO
  - Extending LFT framework to include nonlinear operators
- Demonstrations and Applications
  - Experimental Flight Testbeds
  - Flight Test Programs

There are several research extensions to the μ method that are under investigation. The computation of aeroservoelastic stability margins is performed for an F/A-18 aircraft and results will be published in the 1998 AIAA SDM conference. Methods of reducing the conservatism in the robust margins are being considered using techniques such as wavelet filtering and model updating. The μ method is being extended to account for nonlinear limit cycle oscillations by including nonlinear operators in the LFT framework. Also, the μ method is being applied to several flight test programs to validate and improve the implementation issues.

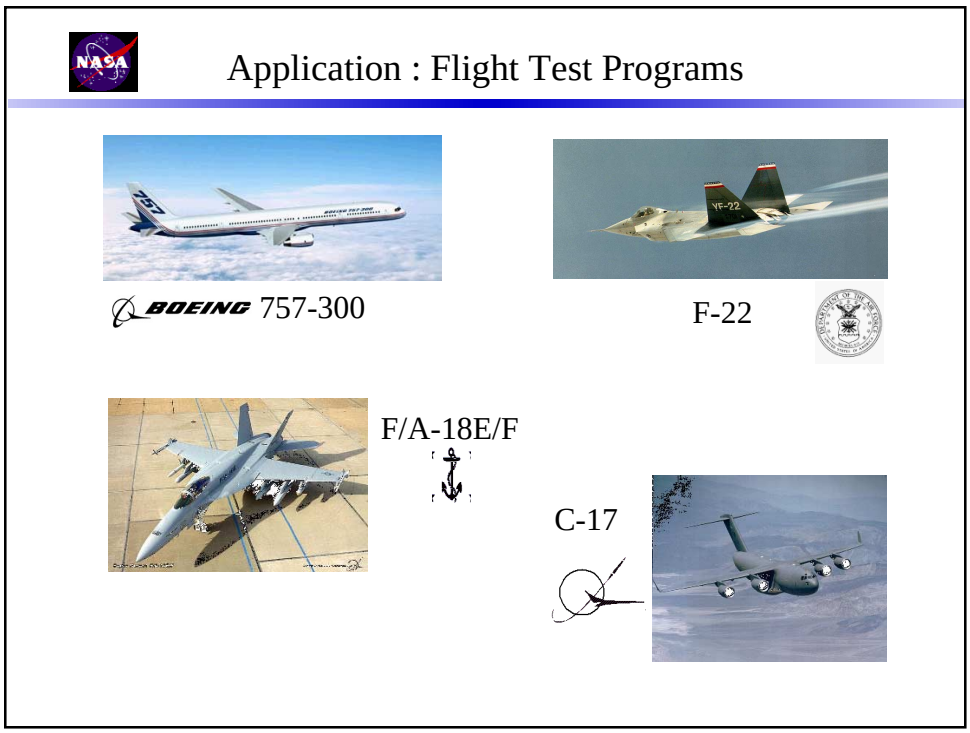
Application : Testbeds

Aerostructures Wing
mounted on F-15 FTF

DAST

Texas A&M University
Aeroelastic System

The μ method is being applied to several small order testbed systems. These systems can be flown at the flutter boundary to ensure the flutterometer predicts the unstable flight conditions.



The μ method is also being considered for several aircraft flight flutter test programs. The cost of these programs could be dramatically reduced with even a small increase in flight test efficiency using the flutterometer.

μ Method : Discussion Topics (general)

- The μ method
  - Why is μ better than traditional/new methods?
  - What do robustness and μ actually mean?
  - How does μ relate to damping?
  - How do I develop an uncertainty description?
  - What level of confidence can I place in these margins?
  - Does μ have any usefulness in analyzing nonlinear systems?
- Incorporating flight data
  - What types of data can be used?
  - What is the model validation actually doing?
  - What are local and global updating schemes?
- Flutterometer
  - What flight test and control room procedures can be used?
  - Can this be done in real-time?
  - What if flutterometer reads “0” margin at my current test point?

μ Method : Discussion Topics (Boeing)

- Control room issues
  - Hardware and software requirements
  - Interaction with other Boeing tools and procedures
  - Work level of the user during a flight test
- Utilizing flutterometer
  - Availability of algorithms
  - Cost and effort to develop and implement
  - External funding possibilities and interests

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13. ABSTRACT (Maximum 200 words)
This paper documents an invited presentation given to The Boeing Company, Seattle, Washington, on September 9, 1997. The audience consisted of structural dynamic and flight test engineers from the Boeing Commercial Airplane Group who were interested in discussing research which may be applied to future flight flutter test programs. A method to compute robust flutter margins is described which is a significant departure from traditional methods. This method uses the structured singular value, μ, to compute a flutter margin which directly accounts for modeling errors such that a worst-case flutter margin is computed with respect to those errors. This method may be applied in several ways. A post-flight application uses data sets from multiple test points to compute worst-case flutter margins and a worst-case flight envelope. An on-line implementation computes flutter margins at each test point to track the flutter margins during a flight test. This on-line implementation is the basis for a flutterometer flight test tool that displays the distance to flutter at a given test point. Such a tool was not previously possible using traditional flutter flight test analysis methods. The F/A-18 System Research Aircraft was used to demonstrate these applications using flight data recorded from test points throughout the flight envelope.

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